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Phases of neutron rich matter and hybrid neutron star in hybrid derivative coupling model

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Abstract : We have studied different phases like liquid gas phase, beta equilibrium phase and deconfined phase involving quarks of neutron rich matter in the frame work of recently proposed hybrid derivative coupling model. We discuss the possible existence of a hybrid star with the core consisting of asymmetric quark matter, an intermediate region of mixed phase containing quarks and neutrinos and a crust of neutrons. We have determined asymmetry parameter (i) β_{infl} for which both pressure and compressibility vanish and (ii) β_{ll} for which binding energy as well as pressure is zero. Baryon density and energy per baryon of self bound asymmetric quark matter resulting from neutron have been calculated Compressibility of asymmetric quark matter has also been determined

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The purpose of this paper is to study different phases and aspects of neutron rich matter or highly asymmetric nuclear matter characterized by asymmetry parameter $\beta = (N-Z)/(N+Z)$ in the framework of recently proposed hybrid derivative coupling model [1]. In the low density region with density $\rho < \rho_0$ (saturation density of symmetric nuclear matter), liquid-gas mixed phase of neutron rich matter can exist only for a very narrow range of β values which are close to unity. We determine $\beta = \beta_{infl}$ close to 1 in the liquid-gas phase region at zero temperature for which binding energy $(\varepsilon/\rho - M)$ has a point of inflection associated with the vanishing of both pressure and bulk modulus K. We also determine $\beta = \beta_{ll}$ for which $(\varepsilon/\rho - M)$ and pressure simultaneously vanish. Further, the particular value of $\beta = \beta_h$ for which binding energy at $\rho = \rho_0$ (which is the density of symmetric matter) vanish has also been evaluated.

It is of interest to study beta equilibrium phase of neutron rich matter at moderately high density. Starting from the usual relation involving chemical potentials μ_n, μ_p and μ_e of neutron, proton and electron respectively in beta equilibrium. We find that the part of the difference $(\mu_n - \mu_p)$ is $4/\beta$ times the symmetry energy which according to Prakash and Anisworth [2] and determines the amount of conversion of some of the neutrons at the top of the Fermi sea into protons, electron and neutrinos.

At sufficiently high density, prevailing in some of the neutron stars, it is possible that neutron matter in the core of a star is transformed into a deconfined phase consisting of quarks. In the first order phase transition of neutron matter to quark matter there is a mixed phase region where there is a coexistence of both neutron matter and asymmetric quark matter which contains down quarks and up quarks in the ratio 2 to 1. In some neutron star with certain central density (partly determined by compressibility) we find that there can exist a hybrid star where the innermost core is occupied by quarks, the intermediate region contains the above mentioned mixed phase and the outermost region the crust of the hybrid star consists of neutrons only. We have evaluated baryon density and energy per baryon of self-bound above mentioned asymmetric quark matter characterized by zero pressure. Compressibility of this quark matter has also been determined.

The mathematical formalism for hybrid derivative scalar coupling in which the ratio of the strength of Yukawa point coupling [3] to that of the derivative coupling of scalar meson to nucleon [4] is taken to be $(1\text{-}\alpha)/\alpha$ has been given in our recent work [1]. In this formalism we work with the following Lagrangian [1] for asymmetric nuclear matter involving fields of nucleon (ψ), vector-isoscalar meson (ω_μ), vector-isovector (ρ_μ) and masses and coupling constants associated with the fields.

$$L=\left(1+\alpha\frac{\sigma g_\sigma}{M}\right)\bar{\psi}\left(i\gamma^\mu\partial_\mu-g_v\gamma^\mu\omega_\mu-\frac{1}{2}g_\rho\gamma^\mu\tau\cdot\rho_\mu\right)\psi-\left[1-(1-\alpha)\frac{\sigma g_\sigma}{M}\right]M\bar{\psi}\psi$$

$$+\frac{1}{2}\left(\partial^\mu\sigma\,\partial_\mu\sigma-m_\sigma^2\sigma^2\right)-\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu}+\frac{1}{2}m_v^2\,\omega^\mu\omega_\mu-\frac{1}{4}\rho^{\mu\nu}\rho_{\mu\nu}+\frac{1}{2}m_\rho^2\cdot\rho^\mu\rho_\mu\,, \quad (1)$$

where

$$M^*=\frac{\left[1-(1-\alpha)\frac{\sigma g_\sigma}{M}\right]}{\left[1+\alpha\frac{\sigma g_\sigma}{M}\right]}M\,. \quad (2)$$

In eq. (1) $\omega_{\mu\nu}$ and $\rho_{\mu\nu}$ are usual tensors. In the mean field theory (MFT) approximation, the field equation for vector-isoscalar field is expressed as

$$\omega_0=\left(\frac{g_v}{m_v^2}\right)\rho \quad (3)$$

where the vector density or baryon density ρ is given by

$$\rho=\sum_{i=n,p}\rho_i=\sum_i\frac{1}{3\pi^2}k_{Fi}^3\,. \quad (4)$$

Similar equations for vector isovector and scalar mesons are found to be

$$\rho_{03}=\frac{1}{2}\left(g_\rho/m_\rho^2\right)(\rho_p-\rho_n) \quad (5)$$

and

$$\sigma(g_\sigma / m_\sigma^2)^{-1}\left[1+\alpha g_s\sigma / M\right]^2 = \sum_i \rho_{si} = \sum \frac{1}{\pi^2}\int_0^{k_{Fi}} M^*(k^2+M^{*2})^{-1/2}\, k^2 dk \,. \tag{6}$$

Fermi momentum k_{Fp} or k_{Fn} can be written as

$$k_{Fp}, k_{Fn} = (1 \mp \beta)^{1/3}\, k_F \,. \tag{7}$$

The relations (2), (6) and (7) help us to determine effective mass M* as a function of density and asymmetry parameter β.

In the following, we introduce some new parameters related to the various coupling constants [4]

$$g_A^2\, M^2 / m_A^2 = C_A^2 = B_A(\rho_0 / M^3)^{-1}, \tag{8}$$

where the suffix A stands for any one of the suffixes 's', 'v' and 'ρ' occurring in eqs. (1) and (2) and $\rho_0 = \frac{2}{3\pi^2}\, k_{F0}^3$ is the saturation density of symmetric nuclear matter.

The energy density and pressure for asymmetric nuclear matter are given by

$$\varepsilon(\rho,\beta) = \sum_{i=n,p} \varepsilon_{Fi} + \varepsilon_\omega + \varepsilon_\rho + \varepsilon_\sigma \tag{9}$$

and

$$P(\rho,\beta) = \sum_{i=n,p} P_{Fi} + \varepsilon_\omega + \varepsilon_\rho - \varepsilon_\sigma \,, \tag{10}$$

where

$$\varepsilon_{Fi} = \frac{1}{\pi^2}\int_0^{k_{Fi}} M^*(k^2+M^{*2})^{1/2}\, k^2\, dk, \tag{11}$$

$$P_{Fi} = (\varepsilon_{Fi} - M^*\rho_{si}) \tag{12}$$

$$(\varepsilon_\omega + \varepsilon_\rho) = \frac{\rho^2}{2M^2}\left(C_v^2 + \frac{1}{4}\beta^2\, C_\rho^2\right), \tag{13}$$

$$\varepsilon_\sigma = \frac{M^4}{2C_s^2}\left[\frac{1-M^*/M}{1-\alpha+\alpha M^*/M}\right]^2 . \tag{14}$$

In the neutron rich matter, some of the neutrons near the top of the Fermi sea undergoes beta decay. In the beta equilibrium chemical potentials of neutron, proton $\mu_{n,p}$ $(= \partial\varepsilon / \partial\rho_{n,p})$ and electron are related by the following equation

$$\mu_n - \mu_p = \mu_e \tag{15}$$

In view of the above relations and charge conservation we have

$$\mu'_n - \mu'_p + \frac{C_\rho^2}{2M^2}\beta\rho = k_{Fp} \,. \tag{16}$$

where effective chemical potentials μ'_n and μ'_p are given by

$$\mu'_{n,p} = \left(k_{Fn,p}^2 + M^{*2}\right)^{1/2} . \tag{17}$$

Expanding in powers of asymmetry parameter β we have

$$\frac{C_\rho^2}{2M^2}\beta\rho + \frac{2}{3}\beta\frac{k_F^2}{\left(k_F^2 + M^{*2}\right)^{1/2}} + \frac{1}{81}\beta^3\frac{k_F^2\left(10k_F^4 + 11k_F^4 + 4M^{*4}\right)}{\left(k_F^2 + M^{*2}\right)^{5/2}} + \ldots$$

$$= k_F(1-\beta)^{1/3} . \tag{18}$$

The combination of the above two terms of the above relation represent $4/\beta$ times the symmetry energy $E_{sym}(\rho)$ as given by Prakash and Ainsworth [2]. Following Bombaci and Lombardo [5] it can easily be seen from (18) that proton fraction in beta equilibrium is approximately given by

$$\frac{1}{2}(1-\beta) = \frac{Z}{N-Z} = \frac{1}{2}\left[\frac{4E_{sym}}{k_F}\right]^3 . \tag{19}$$

It is of interest to study liquid-gas phase and also compressibility of neutron rich matter characterized by the value of the asymmetry parameter β close to 1. For any value of β, the appropriate saturation density is obtained from the condition $P(\rho_{sat}(\beta),\beta) = 0$. Then the corresponding compressibility for the asymmetric system is given by $K_\beta = 9\partial P(\rho,\beta)/\partial\rho\big|_{\rho_{sat}}(\beta)$. Both $\rho_{sat}(\beta)$ and K_β decrease with increasing value of β. For large value of β, the so called parabolic law [5] for the variation of K_β with β is not strictly valid and actual numerical evaluation is needed. In the domain where $\rho < \rho_{sat}(\beta)$, there is an unstable region where the pressure decreases with increasing density. This phenomena is a characteristics of liquid-gas phase transition in nuclear matter. The liquid gas phase transition region like $\rho_{sat}(\beta)$ gets smaller as asymmetry paramcter β increases.

We can determine the value of asymmetry parameter β_{Infl} which is close to 1 and $\rho = \rho_{sat}(\beta_{infl})$ for which both pressure $p(\rho_{sat}(\beta_{infl}),\beta_{infl})$ = 0 and bulk modulus $K_{\beta_{infl}} = 9(\partial p/\partial\rho)\big|_{\rho=\rho_{sat}(\beta_{infl})}$ = 0. It is found that bulk modulus vanishes when $\beta_{infl}(\rho_{sat}(\beta_{infl})) = 0.9015\ (0.275\rho_0)\ 0.915\ (0.190\rho_0)$ and 0.925 $(0.16\rho_0)$ for hybridization parameter α = 0, 1/4 and 1 respectively. The possible curve for binding energy (ε/ρ-M) of asymmetric system *versus* ρ has a point of inflection when $\rho = \rho_{sat}(\beta_{infl})$ and $\beta = \beta_{infl}$. Compressibility K for symmetric nuclear matter is found to be K (in MeV) = 540, 307 and 225 corresponding to $\alpha = 0$, 1/4 and 1 respectively [1]. We have found that K_β (in MeV) = 272, 156 and 13 for $\beta = 0.3$, 0.6 and 0.9 respectively in the case when $\alpha = 1/4$.

Similarly, we have determined the values of $\beta = \beta_{fl}$, $\rho = \rho_{fl}(\beta_{fl})$ for which both binding energy and pressure $P = \rho^2 \partial(\varepsilon/\rho - M)/\partial\rho$ vanish. We have found that $(\beta_{fl}, \rho_{fl}(\beta_{fl})) = (0.79, 0.562\rho_0)$, $(0.82, 0.415\rho_0)$ and $(0.84, 0.32\rho_0)$ when $\alpha = 0$, 1/4 and 1 respectively. We find that $(\varepsilon/\rho - M)$ at $\rho = \rho_0$, for the asymmetric system vanishes for $\beta = 0.7085$, 0.6889 and 0.6888 corresponding to the case when $\alpha = 0$, 1/4 and 1 respectively.

It is of much interest to study highly dense nearly pure neutron matter ($\beta \cong 1$) which exists in the neutron star. At high density there is a possible phase transition from neutron matter to asymmetric quark matter. This may imply possible existence of hybrid star with the core containing quark matter and crust is composed of neutron matter. In view of the above mentioned phenomena, we now consider thermodynamic quantities in the high density region. In this region energy density and pressure of neutron matter are given by

$$\varepsilon_n = \frac{1}{\pi^2}\left[2^{-2/3} k_F^4 + 2^{-4/3} k_F^2 M^{*2} + \ldots\right] + \varepsilon_\omega + \varepsilon_\rho + \varepsilon_\sigma \tag{20}$$

and

$$P_n = \frac{1}{3\pi^2}\left[2^{-2/3} k_F^4 - 2^{-4/3} k_F^2 M^{*2} + \ldots\right] + \varepsilon_\omega + \varepsilon_\rho + \varepsilon_\sigma, \tag{21}$$

where

$$\varepsilon_\sigma + \varepsilon_\rho = (1/2)\rho_0 M B_v' (k_F / k_{F0})^6 \tag{22}$$

and

$$B_v' = B_v + B_\rho / 4, \tag{23}$$

and ε_σ has already been defined by eq. (14)

Effective mass M^* appearing in above equations is given by

$$M^* = 2^{1/3}\pi^2 M / \left[C_s^2 (1-\alpha)^3 k_F^2\right] \quad \text{for } \alpha = 1 \tag{24}$$

$$= 2^{1/12}\left(\pi^2 / C_s^2\right)^{1/4}\left(M^3 / k_F\right)^{1/2} \quad \text{for } \alpha = 1. \tag{25}$$

Chemical potential for purely neutron matter at high density is expressed as

$$\mu_n \cong (k_F / k_{F0})^3 M B_v' + 2^{1/3} k_F + 2^{-3/2} M^{*2} k_F^{-1}. \tag{26}$$

In view of the relation (24) and (26) we can write

$$\frac{k}{k_F} \cong \left(\frac{\mu_n}{MB_v'}\right)^{1/3}\left[1 - \frac{2^{1/3}}{3}\frac{k_{F0}}{\mu_n}\left(\frac{\mu_n}{MB_v'}\right)^{1/3} - \frac{2^{4/3}}{27(1-\alpha)^3 B_s^2}\frac{k_{F0}}{\mu_n}\left(\frac{\mu_n}{MB_v'}\right)^{5/3}\right]. \tag{27}$$

Using (27), we can express P_n as a function of chemical potential

$$P_n = P_n(\mu_n)\,. \tag{28}$$

Chemical potential for quark matter resulting from the deconfinement of neutron matter is expressed as

$$\mu_{(q)} = \mu_u + 2\mu_d\,, \tag{29}$$

where μ_u and μ_d are chemical potentials for u-quark and d-quark respectively and further

$$2\mu_u^3 = \mu_d^3 \tag{30}$$

assuming quarks (u and d) to be mass less.

Pressure, energy density and baryon density for quark matter are given by

$$P_q = \frac{1}{4\pi^2}\mu_q^4 - B, \tag{31}$$

$$\varepsilon_q = 3P_q + 4B, \tag{32}$$

and

$$\rho_{B,q} = \mu_u^3/\pi^2 = (1/\pi^2)(1+2^{4/3})^{-3}\mu_q^3\,, \tag{33}$$

where B is the bag constant in MIT bag model [6] for quarks. In view of the above relations we can write

$$\varepsilon_q = \left(3^{7/3}/4\right)\left(1+2^{4/3}\right)\pi^{2/3}\rho_{B,q}^{4/3}\,. \tag{34}$$

Energy per baryon of above self-bound asymmetric quark matter is

$$\varepsilon_q/\rho_{B,q} = 2^{3/2}\left(1+2^{4/3}\right)^{3/4}\pi^{1/2}B^{1/4}\,. \tag{35}$$

In the following, we give an expression for compressibility of asymmetric quark matter (resulting from deconfinement of neutron matter) K_q evaluated at the density for which $P_q = 0$ corresponding to the self-bound quark matter

$$K_q = 3(2\pi)^{1/2}\left(1+2^{4/3}\right)^{3/4}B^{1/4}\,. \tag{36}$$

Phase transition from neutron matter is determined by the following Gibbs criteria for equality of the chemical potentials and pressures of the two phases

$$\mu_n = \mu_{B,q} = \mu_c \tag{37}$$

and

$$P_n(\mu_n) = P_q(\mu_q) \doteq P_c\,. \tag{38}$$

We have already defined P_n and P_q by (28) and (31) in terms of the corresponding chemical potentials μ_n and μ_q. It is found by actual calculation that μ_c, P_c and other characteristics of the phase transition like normalized baryon density ρ_n/ρ_0 and energy density ε_n and the corresponding quantities $\rho_{B,q}/\rho_0$ and ε_q for quark matter all increase as α increases or bulk modulus (associated with the hybridization parameter α) K decreases. Further all the above characteristics increase with increasing bag constant B. It is found that for almost the same bag constant, μ_c (in MeV) = 1.39, P_c (in MeV fm^{-3}) = 0.150 for the hybrid derivative coupling model characterized by $\alpha = 1/4$ ($B^{1/4} = 178$ MeV) [3] agree very well with the corresponding results 1.37 and 0.149 of Serot and Uechi ($B^{1/4} = 175$ MeV) [7]. In this case, other characteristics of phase transition ρ_n, $\rho_{B,q}$, ε_n and ε_q (in GeV fm^{-3}) are $3.30\rho_0$, $5.13\rho_0$, 0.579 and 0.983 for $\alpha = 1/4$ which generally do not differ very much from the corresponding results $4.02\rho_0$, $6.77\rho_0$, 0.92 and 1.65 of Serot and Uechi [7].

It may noted be that Nishizaki *et al.* [8] have found that in their model characterized by bulk modulus $K = 300$ MeV and maximum mass of neutron star $M_{max} = 1.87\ M_\odot$, the central density designated by ρ_c is $\rho_c = 8.29\rho_0$. We may note that according to the recent estimate of Sharma *et al* [9], bulk modulus $K = 300 \pm 25$ MeV [9]. In our hybrid model characterized by $\alpha = 1/4$, the bulk modulus has almost the same value namely $K = 307$ MeV and it is somewhat likely that ρ_c in this case is about $8\rho_0$. It may be pointed out that we get satisfactory results for bulk properties of symmetric nuclear matter when α is about 1/4. The ρ-meson coupling constant $C_\rho^2 = 104$ for $\alpha = 1/4$. C_s^2 and C_v^2 for $\alpha = 1/4$ have already been given in Ref. [1]. In the above case, we may expect that possible hybrid neutron star may have three regions. In the innermost region starting from central density ρ_c of about $8\rho_0$ to $5.13\rho_0$, we have purely quark matter. In the intermediate region extending from $5.13\rho_0$ to $3.30\rho_0$, we have mixed phase where both neutron matter and asymmetric quark matter coexist. In the outer region or the crust, we have purely neutron matter with density ranging from $3.30\rho_0$ to zero density. The mixed phase of the above mentioned intermediate region of hybrid star is generally characterized by a parameter (called the volume fraction [10] of quark phase) defined by $f_Q = (\rho - \rho_n)/(\rho_{B,q} - \rho_n)$ where ρ is some density of the mixed phase. For bag constant $B^{1/4} = 236$ MeV [11], $\alpha = 1/4$ and $\rho_c = 8\rho_0$, above mentioned ρ_n and $\rho_{B,q}$ for phase transition are found to be $5.32\rho_0$ and $12\rho_0$ respectively and consequently previously mentioned innermost region of pure quark matter does not exist in the hybrid star. It may be pointed out that recently Overgard and Overgard [12] have studied quark star and Mishra *et al.* [13] have considered neutron matter with pion condensate to investigate hybrid quark stars. Possible existence of hybrid star may be inferred from measurements of red shift at the star surface, moment of inertia of the star and neutrino emissivity.

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